3. Derivation of the temperature increase equation:

$\Delta T = 1.66\ \mathbf{\mathit{ln}}\ (C/C_0)$

The assumptions we will make allow us to represent the real atmosphere. This remarkably reasonable representation of the real atmosphere is due in part to the small mean optical thickness of the Earth's atmosphere. "Instant " doubling means there is no feedback from a change in water vapour opacity due to a change in temperature. We assume that the atmosphere is transparent to visible radiation and heating only occurs at the Earth's surface (Grey atmosphere). There is no convection and scattering can be neglected. Finally, we assume local thermodynamic equilibrium. This means that in a localised atmospheric volume below 40kms we consider it to be isotropic (emission is non- directional) with a uniform temperature. Here Kirchhoff's Law is applicable so that emissivity equals absorptivity. Two temperatures (T_e and T_s) are important. The *effective emission temperature* (T_e) is the temperature the Earth would have without an atmosphere just taking into account its reflectivity and its distance from the sun. The flux (F) absorbed by the climate system as:

$$F_e \ = S\,\frac{(1-\alpha_p)}{4} \qquad (19)$$

$where\ \ solar\ constant, S = 1366\ {}^{W}\!/_{m^2}\ \ and\ the\ planetary\ albedo, \alpha_p = 0.32$

Stefan-Boltzmann law for the Earth as a black body (or perfect radiator) gives:

$F = \sigma T^4$

where F is the flux density emitted in W/m^2
σ is the Stefan-Boltzmann constant, and
T is the absolute temperature.

$$F = \sigma T_e^4 \ = \ F_e \qquad (20)$$

T_s is the surface air temperature and $F = \sigma T_s^4$ is intermediate upward flux density (heat) radiated from the surface. ϵ is the fraction of the upward flux (σT_s^4) that is absorbed by the atmosphere and equals that subsequently emitted hence $F = \ \epsilon\sigma T_s^4$

The radiation absorbed in the upper atmosphere at temperature T_a is re-emitted equally in all directions, half upward and half downward. Hence,

$$Hence,\ \ \epsilon\sigma T_s^4 = 2\epsilon\sigma T_a^4\ \ and\ T_a^4 = \ \frac{T_s^4}{2} \qquad (21)$$

$$If\ T_s = 288.15^oK\ then\ T_a = 242.3^oK\ in\ the\ upper\ atmosphere.$$

The flux density out of the top of the atmosphere is given by:

$$F_{out} = \ \epsilon\sigma T_a^4 + (1-\ \epsilon)\sigma T_s^4$$

Parameterisation gives:

$$\Delta F_{out} = \Delta\epsilon(\sigma T_a^4 - \sigma T_s^4) = \Delta\epsilon\sigma\frac{T_s^4}{2} \quad from\ Equation\ 21 \qquad (22)$$

First we calculate the vertical opacity of the atmosphere (τ_g) from the Chamberlain[4] expression that he derived from the general heat transfer equation:

$$T_s^4 = T_e^4\left(1 + \frac{3}{4}\tau_g\right) = T_e^4 A \quad where\ A = 1 + \frac{3}{4}\hat{o}_g \qquad (23)$$

$$T_s^4 = A\,\frac{F_e}{\sigma} = A\,\frac{(1 - \alpha_p)S}{4\sigma}\ from\ Equations\ 19\ and\ 20$$

$$\sigma T_s^4 = A\,\frac{(1 - \alpha_p)S}{4} \qquad (24)$$

We now determine the relation of T_s to ΔF through τ using:

$$\frac{dT_s}{dF} = \frac{dT_s}{d\tau} X \frac{d\tau}{d\epsilon} X \frac{d\epsilon}{dF} \qquad (25)$$

$$\epsilon = 2\left(1 - \frac{1}{1 + \frac{3}{4}\tau_g}\right)$$

$$\frac{d\epsilon}{d\tau} = \frac{3}{2} X \frac{1}{\left(1 + \frac{3}{4}\tau_g\right)^2}\ or\ \frac{d\tau}{d\epsilon} = \frac{2}{3} A^2 \qquad (26)$$

Taking the derivative of Equation (23) and substituting for T_e we have:

$$\frac{dT_s}{d\tau} = \frac{3T_s}{16\left(1 + \frac{3}{4}\tau_g\right)} = \frac{3T_s}{16A} \qquad (27)$$

$$\frac{d\epsilon}{dF} = \frac{2}{\sigma T_s^4} \qquad (28)$$

Substituting Equations 24,26,27 and 28 in Equation 25 we have:

$$\frac{dT_s}{dF} = \frac{3T_s}{16A} X \frac{2}{3} A^2 X \frac{2}{\sigma T_s^4} = \frac{A}{4\sigma T_s^3} = \frac{T_s}{(1 - \acute{a}_p)S} \qquad (29)$$

$$\boldsymbol{\Delta T_s = \frac{T_s}{(1 - \alpha_p)S}\,\Delta F = \frac{T_s}{928.88}\,\Delta F = 0.31\,\Delta F} \qquad (30)$$

Substituting for ΔF from Equation 18, ΔF = 5.35 *ln* (C/C_0)

$$\boldsymbol{\Delta T_s} = \frac{T_s}{(1-\alpha_p)S} X\ 5.35\ \ln(C/C_0) = \frac{\boldsymbol{T_s}}{\mathbf{173.62}} \mathbf{ln}(\boldsymbol{C}/\boldsymbol{C_0}) \quad (31)$$

Substituting for T_S, in Equation 31 gives:

$$\Delta T = 1.66\ ln\ (C/C_0) \quad (32)$$

Greenhouse gases, including carbon dioxide and water vapour, keep the Earth's surface about 33°C warmer than it would otherwise be. How much warming does carbon dioxide itself contribute to the current surface temperature of the Earth? We can calculate the CO_2 flux density (F) at concentration C in the current atmosphere using Equation 16, F = 5*.35 ln* C, from Section 2 above. CO_2 concentration reached 400 ppm on 11th May 2013 and therefore F is 32.05 W/m^2. From Equation 30 we have:

$$\Delta T = 0.31\ \Delta F \quad (33)$$

Therefore, ΔT = 0.31 X 32.05 = **10°C**

Water vapour adds a further 75 W/m^2 giving total ΔF = 107.05 W/m^2. Surface temperature increase ΔT = 0.31 X 107.05 = **33°C**. That is, CO_2 and water vapour increase the surface temperature of the Earth by 33°C.

5. References

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